Confinement of light quarks in QCD at finite density.

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Abstract

The $4q$ effective Lagrangian and the gap equation are derived for light quarks in the confinement phase of QCD. The modification of the confining string due to finite quark density (chemical quark potential μ) is observed. As a surprising result in a multiquark system with a common string junction an attractive well appears of radius μ/σ and of an average depth equal to μ. Possible implications for the density phase transition are discussed.

1 Introduction

Recently the high-density effects in QCD attracted a lot of attention because of the possible density phase transition [1], and interesting accompanying phenomena like Color Superconductivity (CS) [2] (for earlier papers see [3]). On the fundamental level the physical expectation of any phase transition may be connected to the possible reconstruction of the vacuum and for that one needs that the energy density to be of the order of the vacuum energy density $\varepsilon_{cr} \sim \varepsilon_{vac} \approx -\frac{11}{3} N_c \frac{\alpha_s}{32\pi} \langle (F^a_{\mu\nu})^2 \rangle, \varepsilon_{cr} \sim 1$ GeV/fm^3 which provides the drastic change in the vacuum structure and may cause the phase transition. In case of temperature phase transition the corresponding energy density ε is indeed of the order of ε_{cr}, and as it was first argued in [4] and later measured on the lattice [5], the QCD vacuum is strongly transformed in a way, that most part of colorelectric fields evaporate above T_c. In this way T_c was calculated through ε_{cr} and found in good agreement with lattice data [4]. Also, it was shown that in QCD in the framework of hadron resonance gas and making use of the low-energy theorems at $T \neq 0$ [6] the quark condensate and one half (colorelectric component) of gluon condensate evaporate at the same temperature [7], which corresponds to the temperature of quark-hadron phase transition. Besides application of the effective dilaton Lagrangian to gluodynamics [8] and to QCD with light and heavy hadrons [9] permitted to conclude that similar reconstruction of the nonperturbative gluon vacuum takes place at finite temperature.

A similar arguments for the density phase transition would imply that at the baryon density of ~ 1 nucleon/fm^3, i.e. $3 \div 6$ times higher than the standard nuclear density, the vacuum can be reconstructed in such a way, that part of fields, e.g. the colorelectric fields responsible for confinement, disappear above critical density.

A completely different route was trodden by the groups who studied the phenomenon of color superconductivity [2]. Here main argument was the model study of CS in the formalism of the NJL or instanton model (see [10] for a recent review) and the practically important question of coexistence of quark and nuclear matter was also studied lately in the unified NJL approach [11]. In this strategy the question of the vacuum reconstruction is not addressed, however it is usually assumed that confinement disappears in the course of establishing of the CS dynamics. On the other hand the possible importance of vacuum fields for the density transition and CS was suggested in [12], but no detailed theory was presented there. It is the purpose of the present paper to start the investigation of the role of density on the vacuum fields in general and confinement in particular.

In this paper we ask ourselves a short and simple question: how the nonzero baryonic chemical potential μ acts on the confinement of light quarks, and come to the unexpected answer, that the confining string of light quarks is destroyed gradually by μ in such a way, that the one part of string, near the string junction is eaten by the nonzero μ, while at distant r the string survives. The plan of the paper is as follows. In section 2 the effective Lagrangian is derived from the QCD Lagrangian with nonzero μ and their solution is discussed in section 3. Section 4 is devoted to the physical implications of results and prospectives.

2 Derivation of Effective $4q$ Lagrangian

One starts with the QCD partition function in presence of quark chemical potential μ in the Euclidean space-time, and we begin with the zero temperature, $T = 0$.

$$Z = \int DAD\psi D\psi^+ e^{-S_0(A)+\int\ {}^f\psi^+(i\hat\partial+im-i\mu\gamma_4+g\hat A)\ {}^f\psi d^4x} \tag{1}$$

where $S_0(A) = \frac{1}{4}\int (F^a_{\mu\nu}(x))^2 d^4x$, m is the current quark mass (mass matrix $\hat m$ in SU(3)), and the quark operator ${}^f\psi_{a\alpha}(x)$ has flavor index $a(f = 1, ...n_f)$, color index $a(a = 1, ...N_c)$ and Lorenz bispinor index $\alpha(\alpha = 1, 2, 3, 4)$, and we use the contour gauge [13] to express $A_\mu(x)$ in terms of $F_{\mu\nu}$. One has for the contour $z_\mu(s, x)$ starting at point x and ending at $Y = z(0, x)$

$$A_\mu(x) = \int_0^1 ds \frac{\partial z_\nu(s,x)}{\partial s}\frac{\partial z_\rho(s,x)}{\partial x_\mu} F_{\nu\rho}(z(s)) \equiv \int_Y^x d\Gamma_{\mu\nu\rho}(z) F_{\nu\rho}(z). \tag{2}$$

Integrating out the gluonic fields $A_\mu(x)$, one obtains

$$Z = \int D\psi D\psi^+ e^{\int\ {}^f\psi^+(i\hat\partial+im-i\mu\gamma_4)^f\psi d^4x} e^{L^{(2)}_{EQL}+L^{(3)}_{EQL}+\dots} \tag{3}$$

where the EQL proportional to $\langle\langle A^n\rangle\rangle$ is denoted by $L^{(n)}_{EQL}$,

$$L^{(2)}_{EQL} = \frac{g^2}{2}\int d^4x d^4y\ {}^f\psi^+_{a\alpha}(x)\ {}^f\psi_{b\beta}(x)\ {}^g\psi^+_{c\gamma}(y)\ {}^g\psi_{d\varepsilon}(y)\langle A^{(\mu)}_{ab}(x)A^{(\nu)}_{cd}(y)\rangle\gamma^{(\mu)}_{\alpha\beta}\gamma^{(\nu)}_{\gamma\varepsilon} \tag{4}$$

Average of gluonic fields can be computed using (2) as (see [14] for details of derivation)

$$g^2\langle A^{(\mu)}_{ab}(x)A^{(\nu)}_{cd}(y)\rangle = \frac{\delta_{bc}\delta_{ad}}{N_c}\int_0^x du_i\alpha_\mu(u)\int_0^y dv_k\alpha_\nu(v)D^{(E,H)}(u-v)(\delta_{\mu\nu}\delta_{ik}-\delta_{i\nu}\delta_{k\mu}), \quad (5)$$

where $D^{(E,H)}(x)$ is the correlator $\langle E_i(x)E_i(0)\rangle$ or $\langle H_i(x)H_i(0)\rangle$. As it was argued in [14] the dominant contribution at large distances from the static antiquark is given by the color-electric fields, therefore at the first stage we shall write down explicitly $L^{(2)}_{EQL}(el)$ for this case, i.e. taking $\mu=\nu=4$. As a result one has[14]

$$L^{(2)}_{EQL}(el) = \frac{1}{2N_c}\int d^4x\int d^4y\ {}^f\psi^+_{a\alpha}(x)\ {}^f\psi_{b\beta}(x)\ {}^g\psi^+_{b\gamma}(y)\ {}^g\psi_{a\varepsilon}(y)\gamma^{(4)}_{\alpha\beta}\gamma^{(4)}_{\gamma\varepsilon}J^E(x,y) \quad (6)$$

where $J^E(x,y)$ is

$$J^E(x,y) = \int_0^x du_i\int_0^y dv_i D^E(u-v), \quad i=1,2,3. \quad (7)$$

One can form bilinears $\Psi^{fg}_{\alpha\varepsilon}\equiv\ {}^f\psi^+_{a\alpha}\ {}^g\psi_{a\varepsilon}$ and project using Fierz procedure given isospin and Lorentz structures, $\Psi^{fg}_{\alpha\varepsilon}\to\Psi^{(n,k)}(x,y)$. Here we consider only $\psi^+\psi$ bosonization. With the help of the standard bosonization trick (here $\tilde{J}\equiv\frac{1}{N_c}J^E$)

$$e^{-\Psi\tilde{J}\Psi} = \int(\det\tilde{J})^{1/2}D\chi\exp[-\chi\tilde{J}\chi+i\Psi\tilde{J}\chi+i\chi\tilde{J}\Psi] \quad (8)$$

$$Z=\int D\psi D\psi^+ D\chi\exp L_{QML} \quad (9)$$

one obtains the effective Quark-Meson Lagrangian (QML)

$$L^{(2)}_{QML} = \int d^4x\int d^4y\Big\{\ {}^f\psi^+_{a\alpha}(x)[(i\hat{\partial}+im-i\mu\gamma_4)_{\alpha\beta}\delta(x-y)+iM^{(fg)}_{\alpha\beta}(x,y)]\ {}^g\psi_{a\beta}(y)-$$

$$-\frac{1}{N_c}\chi^{(n,k)}(x,y)J^E(x,y)\chi^{(n,k)}(y,x)\Big\} \quad (10)$$

and the effective quark-mass operator is

$$M^{(fg)}_{\alpha\beta}(x,y) = \sum_{n,k}\chi^{(n,k)}(x,y)O^{(k)}_{\alpha\beta}t^{(n)}_{fg}\tilde{J}(x,y). \quad (11)$$

Here the operator $\hat{O}$ is a set of all irreducible combinations of Dirac matrixes.

The QML in Eq.(10) $L^{(2)}_{QML}$ contains functions $\chi^{(n,k)}$ which are integrated out in (9), and the standard way is to find $\chi^{(n,k)}$ from the stationary point of $L^{(2)}_{QML}$. Limiting oneself to the scalar and pseudoscalar fields and using the nonlinear parametrization one can write for the operator $\hat{M}$ in (10)

$$\hat{M}(x,y) = M_S(x,y)\hat{U}(x,y), \hat{U}=exp(i\gamma_5\hat{\phi}), \hat{\phi}(x,y)=\phi^f(x,y)t^f. \quad (12)$$

After integrating out the quark fields one obtains the ECL in the form

$$L^{(2)}_{ECL}(M_S,\hat{\phi}) = -2n_fN_c(J^E(x,y))^{-1}M^2_S(x,y)+N_c tr\log[(i\hat{\partial}+im-i\mu\gamma_4)\hat{1}+iM_S\hat{U}]. \quad (13)$$

The stationary point equations $\frac{\delta L^{(2)}_{ECL}}{\delta M_s} = \frac{\delta L^{(2)}_{ECL}}{\delta \hat{\phi}} = 0$ at $\hat{\phi} = \hat{\phi}_0$, $M_s = M_s^{(0)}$ immediately show that $\hat{\phi}_0 = 0$ and $M_s^{(0)}$ satisfies nonlinear equation

$$iM_S^{(0)}(x,y) = 4trSJ^E(x,y) = (\gamma_4 S\gamma_4)J^E(x,y), \quad S(x,y) = -[i\hat{\partial} + im - i\mu\gamma_4 + iM_S\hat{U}]^{-1}_{x,y}. \tag{14}$$

This equation plays the role of the gap equation and is the main point of our further investigation. For $\mu = 0$ this was done in [14] and in the next section we find how results of [14] are modified by the nonzero μ.

3 The confining string at nonzero μ

Our basic equations (6),(7) are nonlocal in time because of the integral over dx_4dy_4 in (10). This nonlocality and the parameter which it governs can be handled most easily, when one uses instead of $M(z,z')$, $S(z,z')$ the Fourier transforms.

$$S(z_4 - z_4', \mathbf{z}, \mathbf{z}') = \int e^{ip_4(z_4 - z_4')} S(p_4, \mathbf{z}, \mathbf{z}') \frac{dp_4}{2\pi} \tag{15}$$

and the same for $M(z,z')$. Then from (14) one obtains a system of equations

$$(\hat{p}_4 - i\hat{\partial}_z - im + i\mu\gamma_4)S(p_4, \mathbf{z}, \mathbf{w}) - i\int M(p_4, \mathbf{z}, \mathbf{z}')S(p_4, \mathbf{z}', \mathbf{w})d\mathbf{z}' = \delta^{(3)}(\mathbf{z} - \mathbf{w}) \tag{16}$$

To simplify matter, one assumes for $D^E(x)$ the Gaussian form, $D^E(x) = D(0)\exp\left(-\frac{x^2}{4T_g^2}\right)$. Then for $M(p_4, \mathbf{z}, \mathbf{w})$ one has

$$iM(p_4, \mathbf{z}, \mathbf{w}) = 2\sqrt{\pi}T_g \int \frac{dp_4'}{2\pi} e^{-(p_4 - p_4')^2 T_g^2} \times$$
$$\times [J^E(\mathbf{z}, \mathbf{w})\gamma_4 S(p_4', \mathbf{z}, \mathbf{w})\gamma_4] \tag{17}$$

where J^E is defined in (7) and we have factored out the time–dependent exponent, using the Gaussian representation of $D(u)$.

All dependence of M on p_4 as can be seen in (17) is due to the factor $\exp[-(p_4 - p_4')^2 T_g^2]$ and disappears in the limit when T_g goes to zero, while the string tension $\sigma \sim D(0)T_g^2$ is kept fixed. This limit can be called the string limit of QCD, and we shall study its consequences for equations (16),(17) in this section.

So in the string limit, with M independent of p_4, let us consider the Hermitian Hamiltonian

$$\hat{H}\psi_n \equiv (\frac{\alpha_i}{i}\frac{\partial}{\partial z_i} + \beta m - \mu)\psi_n(\mathbf{z}) + \beta\int M(p_4 = 0, \mathbf{z}, \mathbf{z}')\psi_n(\mathbf{z}')d^3\mathbf{z}' = \tilde{\varepsilon}_n(\mu)\psi_n(\mathbf{z}) \tag{18}$$

with eigenfunctions ψ_n satisfying usual orthonormality condition

$$\int \psi_n^+(x)\psi_m(x)d^3x = \delta_{nm},$$

From (18) it is clear, that one can redefine $\tilde{\varepsilon}_n(\mu)+\mu\equiv\varepsilon_n$, and ε_n and ψ_n do not depend on μ. Therefore in all subsequent formulas one can use the same equations as in [14], but with the replacement $\varepsilon_n\to\varepsilon_n-\mu$. In particular, the Green's function S can be expressed as

$$S(p_4,\mathbf{x},\mathbf{y})=\sum_n\frac{\psi_n(\mathbf{x})\psi_n^+(\mathbf{y})}{p_4\gamma_4-i(\varepsilon_n-\mu)\gamma_4} \tag{19}$$

Inserting (19) into (17) one has integrals of the type:

$$\int_{-\infty}^{\infty}\frac{dp'_4}{2\pi}\frac{e^{-(p_4-p'_4)^2T_g^2}}{(p'_4\gamma_4-i(\varepsilon_n-\mu)\gamma_4)}=\frac{i}{2}\gamma_4 sign(\varepsilon_n-\mu)(1+0(p_4T_g,|\varepsilon_n|T_g) \tag{20}$$

Note, however, that the result depends on the boundary conditions. If, e.g., one imposes the causality–type boundary condition, then one obtains

$$\int\frac{dp'_4}{2\pi}\frac{e^{ip'_4h_4}}{\gamma_4(p'_4-i(\varepsilon_n-\mu))}=\left\{\begin{array}{ll} i\gamma_4e^{-\varepsilon h_4}\theta(\varepsilon_n-\mu), & h_4>0\\ -i\gamma_4\theta(\mu-\varepsilon_n)e^{\varepsilon h_4}, & h_4<0\end{array}\right.$$

We are thus led to the following expression for M in the string limit

$$M(p_4=0,\mathbf{z},\mathbf{w})=\sqrt{\pi}T_gJ^E(\mathbf{z},\mathbf{w})\gamma_4\Lambda(\mathbf{z},\mathbf{w}) \tag{21}$$

where the definition is used

$$\Lambda(\mathbf{z},\mathbf{w})=\sum_n\psi_n(\mathbf{z})sign(\varepsilon_n-\mu)\psi_n^+(\mathbf{w}) \tag{22}$$

Let us disregard for the moment the possible appearance in M of the vector component (proportional to $\gamma_\mu,\mu=1,2,3,4$) and concentrate on the scalar contribution only, since that is responsible for CSB and confinement. Then one can look for solutions of the Dirac equation (18) in the following form [14]

$$\psi_n(\vec{r})=\frac{1}{r}\left(\begin{array}{c} G_n(r)\Omega_{jlM}\\ iF_n(r)\Omega_{jl'M}\end{array}\right) \tag{23}$$

where $l'=2j-l$, and introducing the parameter $\kappa(j,l)=(j+\frac{1}{2})sign(l-j)$, and replacing M by a local operator (the generalization to the nonlocal case is straightforward but cumbersome, for a possible change in the nonlocal case see [14]). We obtain a system of equations

$$\left\{\begin{array}{l} \frac{dG_n}{dr}+\frac{\kappa}{r}G_n-(\varepsilon_n-\mu+m+M(r))F_n=0\\ \frac{dF_n}{dr}-\frac{\kappa}{r}F_n+(\varepsilon_n-\mu-m-M(r))G_n=0\end{array}\right. \tag{24}$$

Eq.(23) possesses a symmetry $(\varepsilon_n-\mu,G_n,F_n,\kappa)\leftrightarrow(\mu-\varepsilon_n,F_n,G_n,-\kappa)$ which means that for any solution of the form (22) corresponding to the eigenvalue $\varepsilon_n-\mu$, there is another solution of the form

$$\psi_{\mu-\varepsilon_n}(r)=\frac{1}{r}\left(\begin{array}{c} F_n(r)\Omega_{jl'M}\\ iG_n(r)\Omega_{jlM}\end{array}\right) \tag{25}$$

corresponding to the eigenvalue $(\mu-\varepsilon_n)$.

Therefore the difference, which enters (22) can computed in terms of F_n, G_n as follows

$$\Lambda(\mathbf{z},\mathbf{w}) = \Lambda_0(\mathbf{z},\mathbf{w}) - \Delta\Lambda(\mathbf{z},\mathbf{w}) \tag{26}$$

where Λ_0 is the value of Λ for $\mu = 0$, i.e. the same as in [14], while $\Delta\Lambda$ is defined as

$$\Delta\Lambda(\mathbf{z},\mathbf{w}) = 2\sum_{0<\varepsilon_n<\mu} \psi_n(\mathbf{z})\psi_n^+(\mathbf{w}). \tag{27}$$

Using decomposition (23) one can write $\Delta\Lambda$ as

$$\Delta\Lambda(\mathbf{r},\mathbf{r}') = 2\sum_{0<\varepsilon_n,\mu} \begin{pmatrix} G_n G_n^+ \Omega\Omega^+, & -iG_n F_n^+ \Omega\Omega'^+ \\ iF_n G_n^+ \Omega'\Omega^+, & F_n F_n^+ \Omega'\Omega'^+ \end{pmatrix} \tag{28}$$

where we have denoted $\Omega \equiv \Omega_{jlM}, \Omega' = \Omega_{jl'M}$, and we disregard nondiagonal part of Λ.

At this point one can follow the relativistic WKB method for Dirac equation [15] applied to calculation of Λ in [14] in case of $\mu = 0$. The classically available region for $\psi_n(\mathbf{r})$ with energy $\varepsilon_n \equiv \varepsilon$ is $r_{\min} \le r \le r_{\max}$, where $r_{\max,\min} = \frac{\varepsilon^2 \pm\sqrt{\varepsilon - 4\sigma^2\kappa^2}}{2\sigma^2}$, and the summation over n in (27) transforms into integration over $d\varepsilon$, with the lower limit (for a given r) $\varepsilon_{\min} = \sigma r$. In this way one has for the upper diagonal element in (28).

$$\Delta\Lambda(+,+) = \frac{2\sigma}{\pi^2 r}\delta(1-\cos\theta_{\mathbf{rr}'})\int_1^{\mu/\sigma r} d\tau \frac{\tau+1}{\sqrt{\tau^2-1}}\cos(a\sqrt{\tau^2-1})\theta(\mu-\sigma r) \tag{29}$$

with $a = \sigma r|r-r'|$, and we keep $r \approx r'$ everywhere except for a, since for large a (when r is far from r') both Λ and $\Delta\Lambda$ fast decrease.

In a similar way for the lower diagonal element in (28) one has

$$\Delta\Lambda(-,-) = \frac{2\sigma}{\pi^2 r}\delta(1-\cos\theta_{\mathbf{rr}'})\int_1^{\mu/\sigma r} d\tau \frac{\tau-1}{\sqrt{\tau^2-1}}\cos(a\sqrt{\tau^2-1})\theta(\mu-\sigma r) \tag{30}$$

and taking Λ_0 in (25) from [14] the resulting form for Λ (26) is

$$\Lambda(\mathbf{r},\mathbf{r}') \equiv \beta\Lambda_{scalar} + \hat{1}\Lambda_{vector} = \frac{\beta\sigma}{\pi^2 r}\delta(1-\cos\theta_{\mathbf{rr}'})\int_{\mu/\sigma r}^{\infty} \frac{d\tau\cos(a\sqrt{\tau^2-1})}{\sqrt{\tau^2-1}} -$$

$$- \hat{1}\frac{2\sigma}{\pi^2 r}\delta(1-\cos\theta_{\mathbf{rr}'})\int_1^{\mu/\sigma r}\frac{\tau d\tau}{\sqrt{\tau^2-1}}\cos(a\sqrt{\tau^2-1}). \tag{31}$$

Here $\beta \equiv \gamma_4$, and $\hat{1}$ is the unit Dirac matrix, which means, that the second term on the r.h.s. of (31) contributes to the vector part of the resulting mass operator (3), while the first term contributes to the scalar part. One should take into account, that in the first term,

$$\Lambda_{scalar} = \frac{\sigma}{\pi^2 r}\delta(1-\cos\theta_{\mathbf{rr}'})\int_{\tau_{\min}(\mu)}^{\infty}\frac{d\tau\cos(a\sqrt{\tau^2-1})}{\sqrt{\tau^2-1}} \tag{32}$$

$\tau_{min}(\mu) = \mu/\sigma r$ for $\mu > \sigma r$ and 1 otherwise, so that for large $r, r \gg \frac{\mu}{\sigma}$, one has the standard μ-independent value

$$\Lambda_{scalar}(r \sim r' > \mu/\sigma) = \frac{\sigma}{\pi^2 r}K_0(a)\delta(1-\cos\theta_{\mathbf{rr}'}) \tag{33}$$

where we have used relations for the McDonald function K_0

$$K_0(a) = \int_0^\infty \frac{\cos ax dx}{\sqrt{1+x^2}}, \quad \int_0^\infty da K_0(a) = \frac{\pi}{2}. \tag{34}$$

One can check that at large $r, r'(r \sim r' > \mu/\sigma)$ $\Lambda_{scalar} \approx \Lambda^{(\mu=0)}_{scalar}$ is a smeared δ -function,

$$\int \Lambda^{(\mu=0)}_{scalar}(\mathbf{r}, \mathbf{r}') d^3\mathbf{r}' = 1. \tag{35}$$

However for large $\mu, \mu \gg \sqrt{\sigma}$, Λ_{scalar} is different from $\Lambda_{scalar}(\mu = 0)$, and for $r \sim r' < \mu/\sigma$ one has approximately

$$\Lambda_{scalar}(\mathbf{r}, \mathbf{r}') = \Lambda^{(\mu=0)}_{scalar} - f(r, r')\theta(\mu - \sigma r) \tag{36}$$

where

$$f(r, r') = \int_1^{\mu/\sigma r} \frac{d\tau \cos(a\sqrt{\tau^2 - 1})}{\sqrt{\tau^2 - 1}} = \int_1^{\lambda_0} \frac{d\lambda}{\sqrt{1+\lambda^2}} \cos a\lambda, \tag{37}$$

and $\lambda_0 = \sqrt{\left(\frac{\mu}{\sigma r}\right)^2 - 1}$.

Let us now turn to the vector part of interactions, Λ_{vector},

$$\Lambda_{vector} = -\frac{2\sigma}{\pi^2 r} \delta(1 - \cos\theta_{\mathbf{rr}'}) \int_1^{\mu/\sigma r} \frac{\tau d\tau}{\sqrt{\tau^2 - 1}} \cos(a\sqrt{\tau^2 - 1})\theta(\mu - \sigma r) =$$

$$-\hat{1} \frac{2\sigma}{\pi^2 r} \delta(1 - \cos\theta_{\mathbf{rr}'}) \frac{\sin(a\sqrt{\left(\frac{\mu}{\sigma r}\right)^2 - 1})}{a}. \tag{38}$$

Returning back to Eq. (21) one can deduce, that

$$M(p_4 = 0, \mathbf{r}, \mathbf{r}') = \sqrt{\pi} T_g J^E(\mathbf{r}, \mathbf{r}')[\Lambda_{scal}(\mathbf{r}, \mathbf{r}') + \gamma_4 \Lambda_{vector}] \equiv M_{scal} + \gamma_4 M_{vect.} \tag{39}$$

Taking into account, that at $r, r' \gg T_g$ and for the Gaussian $D^E(x)$ one has from (7)

$$J^E(\mathbf{r}, \mathbf{r}') \cong \frac{(\mathbf{rr}')}{rr'} 2T_g \sqrt{\pi} D(0) \min(r, r') \tag{40}$$

one has for M at $r, r' \gg T_g$ and for $r \cong r'$

$$M_{scal}(r, r') = \sigma r(\tilde{\delta}^{(3)}(\mathbf{r}, \mathbf{r}') - \xi(\mathbf{r}, \mathbf{r}')) \tag{41}$$

where $\tilde{\delta}^{(3)}(\mathbf{r}, \mathbf{r}') \equiv \Lambda^{(\mu=0)}_{scal}(\mathbf{r}, \mathbf{r}')$ and

$$\xi(\mathbf{r}, \mathbf{r}') = \delta_\mu = \frac{\sigma}{\pi^2 r}(1 - \cos\theta_{\mathbf{rr}'}) f(r, r')\theta(\mu - \sigma r) \tag{42}$$

and $f(r, r')$ is defined in (37).

For M_{vect} one has, using (38),

$$M_{vect}(\mathbf{r}, \mathbf{r}') = -2\sigma r \varphi_\mu(\mathbf{r}, \mathbf{r}')\theta(\mu - \sigma r) \tag{43}$$

where

$$\varphi_\mu(\mathbf{r},\mathbf{r}') = \frac{\sigma}{\pi^2 r}\delta(1-\cos\theta_{\mathbf{r}\mathbf{r}'})\frac{\sin(a\sqrt{\left(\frac{\mu}{\sigma r}\right)^2-1})}{a}. \tag{44}$$

Note, that one should actually symmetrize all these expressions, e.g. $\frac{1}{r}\to\frac{1}{\sqrt{rr'}}$ etc., but we always are in the regime, where $r\approx r'$. To estimate the magnitude of nonlocal kernel $M_{scal}(\mathbf{r},\mathbf{r}')$ and $M_{vect}(\mathbf{r},\mathbf{r}')$ it is convenient to introduce in (18) the local limit of the mass operator, namely

$$\bar{M}_{scal,vect}(r) = \int d^3\mathbf{r}' M_{scal,vect}(\mathbf{r},\mathbf{r}'). \tag{45}$$

Exploiting the equalities

$$\int d^3\mathbf{r}'\tilde{\delta}^{(3)}(\mathbf{r},\mathbf{r}') = \int d^3\mathbf{r}'\xi(\mathbf{r},\mathbf{r}') = \int d^3\mathbf{r}'\varphi_\mu(\mathbf{r},\mathbf{r}') = 1 \tag{46}$$

one arrives at the expressions

$$\bar{M}_{scal}(r) = \sigma r\theta(\sigma r-\mu), \quad \bar{M}_{vect} = -2\sigma r\theta(\mu-\sigma r). \tag{47}$$

In the next section we shall discuss approximations made in deriving (41), (43), (47) and physical implications of these results.

4 Discussion of results

Results of the previous section Eqs. (41), (43), (47), can be formulated as follows. The relativistic WKB analysis leads to the μ-dependent modification of the confining string, where the piece $[0,\mu/\sigma]$ of the string near the origin of the string (situated at the heavy quark position in case of heavy-light quark, or at the string junction position in the case of baryons), is dissolved, and the linear confinement starts beyond the critical radius $r_{cr}=\mu/\sigma$. Moreover, an attractive vector interaction appears in the same interval with the average magnitude $\langle\bar{M}_{vect}\rangle\sim\mu$.

These conclusions should be taken as qualitative. First of all, the WKB method is not a good approximation at small distances, and we have omitted exponentially damped part of $\psi_n(\mathbf{z})$ in the spectrum, therefore the inner part of the string is to some extent delocalized (see [14] for details) and smoothed.

Secondly, we have not taken into account a possible modification and destruction of the vacuum due to the influence of high density quark matter, which might decrease σ or cancel the string completely (as it is happens in the thermal phase transition [4]). The phenomenon of this kind was observed on the lattice [16] where deconfinement temperature decreased under the influence of applied external Abelian field.

If however, no density induced vacuum deconstruction takes place, then the resulting physical picture according to Eqs. (47), is the net decreasing of confinement in the inner region of some ensemble of quarks, and appearance of attractive vector potential of the order of μ acting on each quark. This may cause creation of deconfined bubbles consisting of $3n$ quarks, $n=2,3,..$ in the midst of the nuclear medium, and dynamically is similar to the $3nq$ bag formation, which was studied before in the framework of the Quark Compound Bag model [17]. Note, however, that bag boundary conditions might be strongly modified as compared to the standard MIT bag model. A quantitative analysis of this situation needs a more accurate analysis of the $3n$ quark system using nonlinear equations for the $3nq$ Green's function, generalizing Eq. (14).

The formation of these high-density $3nq$ bubbles may be connected with the explanation so-called cumulative effects in the hadron-nucleus(and nucleus-nucleus) collisions, for an example of this discussion see [18] and refs. therein.

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